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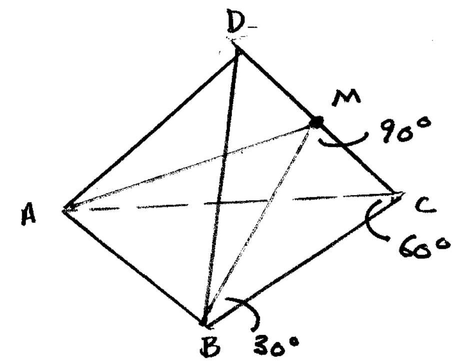
[1.1]

Since ΔACD is one face of tetrahedro, $\angle A = \angle C = \angle D = 60°$.
M midpt of CD creates ΔAMC, a $30°-60°-90°$ triangle, with $\angle M = 90°$.

So $\vec{AM} \cdot \vec{CD} = 0$.

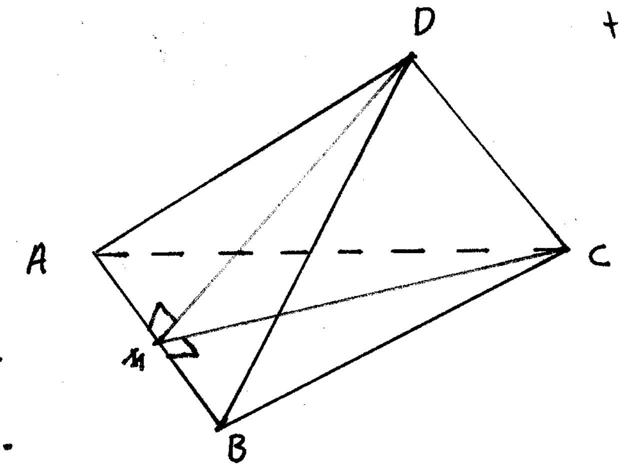
Similar reasonins leads to

$$\vec{BM} \cdot \vec{CD} = 0$$



[1.2] **SOLUTION 1**

ABCD regular tetrahedron



Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be Position vectors of A, B, C, D.

Let M be midpoint of AB.

Then MC ⊥ AB and MD ⊥ AB, because $\triangle MBC$ and $\triangle MBD$ both 30°–60°–90°. So $\vec{MC} \cdot \vec{AB} = 0$ and $\vec{MD} \cdot \vec{AB} = 0$.

This means

$$(\vec{c} - \vec{m}) \cdot \vec{AB} = 0 \qquad \text{(EQ1)}$$

$$(\vec{d} - \vec{m}) \cdot \vec{AB} = 0 \qquad \text{(EQ2)}$$

Subtracting EQ2 from EQ1

$$\left[(\vec{c} - \vec{m}) \cdot \vec{AB}\right] - \left[(\vec{d} - \vec{m}) \cdot \vec{AB}\right] = 0$$

$$\left[(\vec{c} - \vec{m}) - (\vec{d} - \vec{m})\right] \cdot \vec{AB} = 0$$

$$\left[\vec{c} - \vec{m} - \vec{d} + \vec{m}\right] \cdot \vec{AB} = 0$$

$$(\vec{c} - \vec{d}) \cdot \vec{AB} = 0$$

$$\vec{CD} \cdot \vec{AB} = 0$$

Therefor CD ⊥ AB.

□

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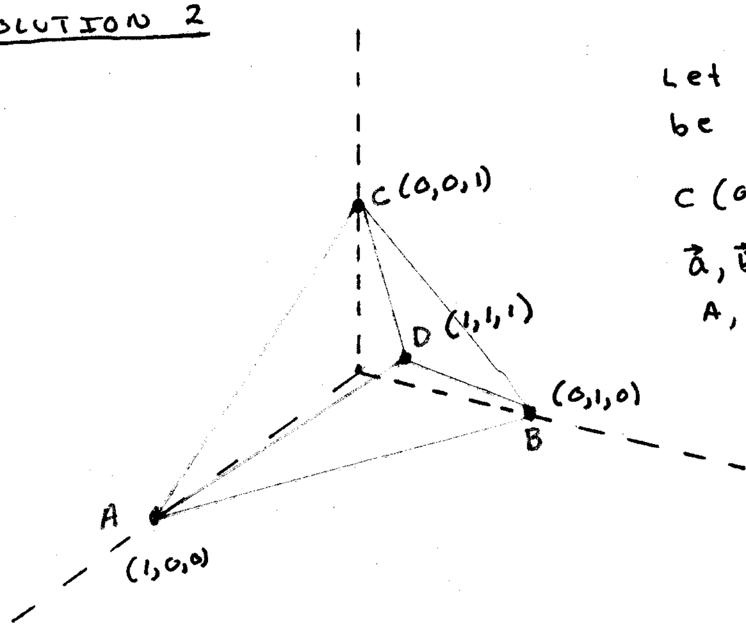
[1.2] SOLUTION 2



Let vertices of tetrahedron be $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$, $D(1,1,1)$ as shown.

$\vec{a}, \vec{b}, \vec{c}, \vec{d}$ position vectors of A, B, C, D.

$$\vec{AB} = \vec{b} - \vec{a} = \langle 0,1,0 \rangle - \langle 1,0,0 \rangle = \langle -1,1,0 \rangle$$

$$\vec{CD} = \vec{d} - \vec{c} = \langle 1,1,1 \rangle - \langle 0,0,1 \rangle = \langle 1,1,0 \rangle$$

$$\vec{AB} \cdot \vec{CD} = \langle -1,1,0 \rangle \cdot \langle 1,1,0 \rangle$$

$$= -1 + 1 + 0$$

$$= 0$$

$\therefore \quad AB \perp CD$

$\square$

[2.1] $P_0\ (2, -3, 7)$. $\vec{u} = \langle 1, 1, -4 \rangle$.

$$x - 2 = y + 3 = \frac{7 - z}{4}$$

[2.2] $$2x - 6 = 4 - y = z - 5$$

$$2(x - 3) = 4 - y = z - 5$$

$$x - 3 = \frac{4 - y}{2} = \frac{z - 5}{2}$$

$$x - 3 = \frac{y - 4}{-2} = \frac{z - 5}{2}$$

$\therefore$ direction vector is $\langle 1, -2, 2 \rangle$

[2.3] θ = angle of line of [2.1] and line of [2.2]

$$\cos\theta = \frac{\langle 1, 1, -4 \rangle \cdot \langle 1, -2, 2 \rangle}{|\langle 1, 1, -4 \rangle| |\langle 1, -2, 2 \rangle|}$$

$$= \frac{-9}{3\sqrt{2} \cdot 3}$$

$$= -\frac{1}{\sqrt{2}}$$

$$= \frac{-\sqrt{2}}{2}$$

$\therefore\ \theta = 135°$

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[3.1] $\ell \quad \frac{x-1}{2} = \frac{2-y}{3} = z+2$

$\alpha \quad 3x - y - 2z = 12$

$$\ell \begin{cases} \frac{x-1}{2} = t \Rightarrow x = 2t+1 \\ \frac{2-y}{3} = t \Rightarrow y = 2-3t \\ z+2 = t \Rightarrow z = t-2 \end{cases}$$

{parametric EQNS of ℓ}

$3(2t+1) - (2-3t) - 2(t-2) = 12$

$6t + 3 - 2 + 3t - 2t + 4 = 12$

$7t + 5 = 12$

$\boxed{t = 1}$ {when $t=1$, ℓ intersects α}

Then point of intersection is

$x = 2(1) + 1 = 3$

$y = 2 - 3(1) = -1$

$z = 1 - 2 = -1$

$\therefore$ $P(3, -1, -1)$ is point of intersection.

{It is easy to check this by substitution into EQNS for ℓ and α.}

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[3.2] Direction vector of ℓ is $\vec{u} = \langle 2, -3, 1 \rangle$

$\vec{n}$ of $\alpha = \langle 3, -1, -2 \rangle$

$$\cos\theta = \frac{\langle 2,-3,1\rangle \cdot \langle 3,-1,-2\rangle}{|\vec{u}||\vec{n}|}$$

$$= \frac{7}{\sqrt{14}\sqrt{14}}$$

$$= \frac{1}{2}$$

$\therefore \ \theta = 60°$

[4.1] $x = 5$

[4.2] $-2(x-3) + 2(y+2) - 3(z-5) = 0$

$-2x + 6 + 2y + 4 - 3z + 15 = 0$

$\therefore \ -2x + 2y - 3z + 25 = 0$

[4.3] $3(x-4) + 6(y+2) - 4(z-3) = 0$

$3x + 6y - 4z - 12 + 12 + 12 = 0$

$\therefore \ 3x + 6y - 4z + 12 = 0$

[4.4] $\vec{a} = \langle 0,4,0\rangle - \langle 3,0,0\rangle = \langle -3,4,0\rangle$

$\vec{b} = \langle 0,0,5\rangle - \langle 3,0,0\rangle = \langle -3,0,5\rangle$

$$\vec{n} = \begin{vmatrix} i & j & k \\ -3 & 4 & 0 \\ -3 & 0 & 5 \end{vmatrix} = 20i + 15j + 12k$$

Plane: $20(x-0) + 15(y-4) + 12(z-0) = 0$

$\therefore \ 20x + 15y + 12z - 60 = 0$

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[5] α: $3x + z - 1 = 0$

β: $x - \sqrt{5}\,y + 2z = 0$

$\vec{n}_\alpha = \langle 3, 0, 1 \rangle$

$\vec{n}_\beta = \langle 1, -\sqrt{5}, 2 \rangle$

$$\cos\theta = \frac{\langle 3,0,1\rangle \cdot \langle 1, -\sqrt{5}, 2\rangle}{|\langle 3,0,1\rangle|\,|\langle 1,-5,2\rangle|}$$

$$= \frac{5}{\sqrt{10}\,\sqrt{10}}$$

$$= \frac{1}{2}$$

$\therefore\ \theta = 60^\circ$

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[6] <u>Solution 1</u> (uses cross product)

$P_0\,(-2, 1, 3)$

$\alpha :\ x - y + z = 0$ $\qquad \vec{n}_\alpha = \langle 1, -1, 1 \rangle$

$\beta :\ 2x + 3y - z = 5$ $\qquad \vec{n}_\beta = \langle 2, 3, -1 \rangle$

Desire plane $\gamma \perp \alpha$, $\gamma \perp \beta$, P_0 in γ, normal vector $\vec{n}_\gamma$

$$\vec{n}_\gamma = \vec{n}_\alpha \times \vec{n}_\beta$$

$$= \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \langle -2, 3, 5 \rangle$$

then

$$\gamma :\quad -2(x+2) + 3(y-1) + 5(z-3) = 0$$

$$-2x + 3y + 5z - 4 - 3 - 15 = 0$$

$$2x - 3y - 5z + 22 = 0$$

$$\therefore\quad \gamma :\quad 2x - 3y - 5z + 22 = 0$$

$\square$

[6] Solution 2

$P_0\,(-2,1,3)$

$\alpha \quad x - y + z = 0 \qquad \vec{n}_\alpha = \langle 1,-1,1\rangle \qquad \vec{n}_\gamma = \langle a,b,c\rangle$

$\beta \quad 2x + 3y - z = 5 \qquad \vec{n}_\beta = \langle 2,3,-1\rangle$

$\gamma: \quad a(x+2) + b(y-1) + c(z-3) = 0$

$\gamma \perp \alpha \Rightarrow \vec{n}_\alpha \cdot \vec{n}_\gamma = 0 \Rightarrow \langle 1,-1,1\rangle \cdot \langle a,b,c\rangle = 0 \Rightarrow a - b + c = 0$

$\gamma \perp \beta \Rightarrow \vec{n}_\beta \cdot \vec{n}_\gamma = 0 \Rightarrow \langle 2,3,-1\rangle \cdot \langle a,b,c\rangle = 0 \Rightarrow 2a + 3b - c = 0$

$$\begin{array}{r} a - b + c = 0 \\ 2a + 3b - c = 0 \\ \hline 3a + 2b = 0 \end{array}$$

Let $a = t,\ t \in \mathbb{R}$

then $b = -\frac{3}{2}t$

$c = -\frac{5}{2}t$

Since we are interested only in the direction of $\langle a,b,c\rangle$,
choose a convienent value for t; e.g. $t = 2$

$t = 2 \Rightarrow \langle a,b,c\rangle = \langle 2,-3,-5\rangle$.

Then

$\gamma: \quad 2(x+2) - 3(y-1) - 5(z-3) = 0$

$2x - 3y - 5z + 4 + 3 + 15 = 0$

$\therefore\ \gamma: \quad 2x - 3y - 5z + 22 = 0$

$\square$